TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 6 2020

Proof and Complex Numbers

STUDENT'S NAME _____

DATE: Wednesday 9th September **TIME:** 50 minutes **MARKS**: 52

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Scientific Calculator only, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (2 marks)

Express the following recurring decimal as a fraction. It is not necessary to simplify the fraction.

$0.0\overline{13}$

2. (4 marks)

Prove by contradiction $\sqrt{3}$ is irrational.

3. (4 marks)

Prove the sum of five consecutive odd numbers is a multiple of five.

4. (12 marks)

Given $z = 4 + 3i$ and $w = 2 - 5i$ determine:

(a) w^2 [2]

(b) $z\overline{w}$ [2]

(c) $\frac{w}{z}$ [3]

(d) $3z - 4w$ [2]

(e) $Im\left(\frac{1}{z}\right)$ [3]

5. (4 marks)

A quadratic equation in the form $x^2 + bx + c = 0$ has one of its roots $7 - 3i$.
Determine *b* and *c*.

6. (6 marks)

Prove

(a) $n^3 - n$ is a multiple of 6 , for $n \geq 2$ [3]

(b) $\overline{wz} = \overline{w}\,\overline{z}$ given w and z are complex numbers [3]

7. (8 marks)

(a) Solve $x^2 - 10x + 29 = 0$ [4]

(b) Determine the complex number z given $z - 2\bar{z} = 5 + 6i$ [4]

8. (6 marks)

Prove the following conjecture using mathematical induction,

for all $n \geq 1$, $\frac{x^{n+1} - 1}{x - 1} = 1 + x + x^2 + ... + x^n$ where $x \neq 1$

9. (6 marks)

Use mathematical induction to prove the following conjecture.

$$2^{n+1}\sin x \cos x \cos(2x)\cos(4x)\ldots\cos(2^n x) = \sin(2^{n+1}x) \quad \text{for} \quad n \geq 0,\ n \in \mathbb{Z}$$